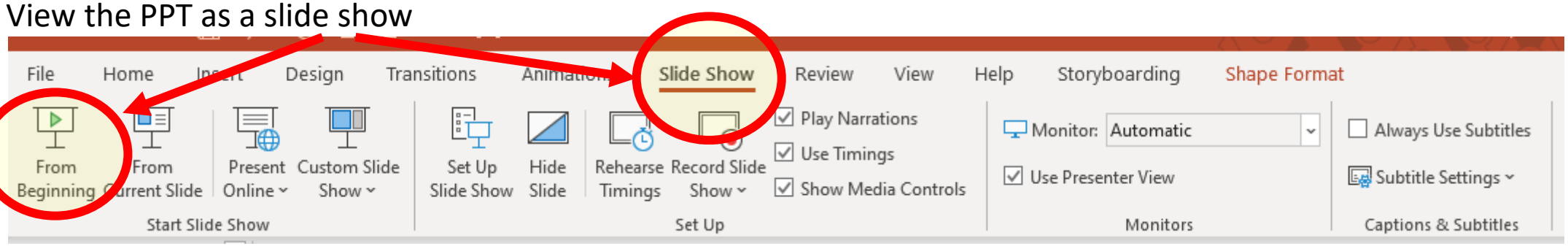
How to best use these slides…

- View the PPT as a slide show



- Then click through every step
  - Mouse clicks will advance the slide show
  - Left/right arrow keys move forward/backward
  - Mouse wheel scrolling moves forward/backward
- When a question is posed, stop and think it through, try to answer it yourself before clicking
- If you have questions, email me, ask in the Teams Student Center channel!

LESSON 6.6a

Solving Exponential Equations

Today you will:

- Solve exponential equations.

- Practice using English to describe math processes and equations

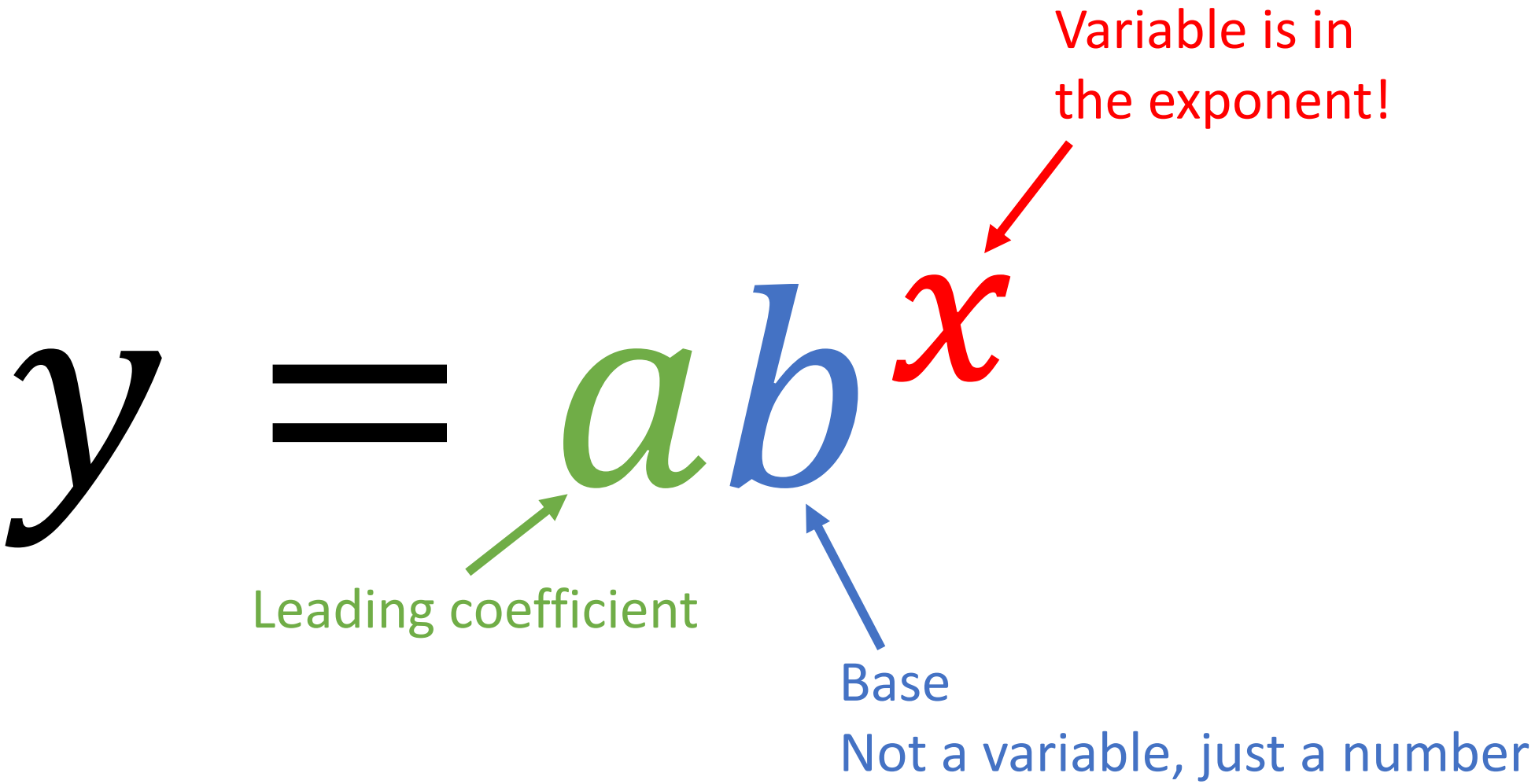
Core Vocabulary:

- Exponential Equations, p. 334

Previous Vocabulary:

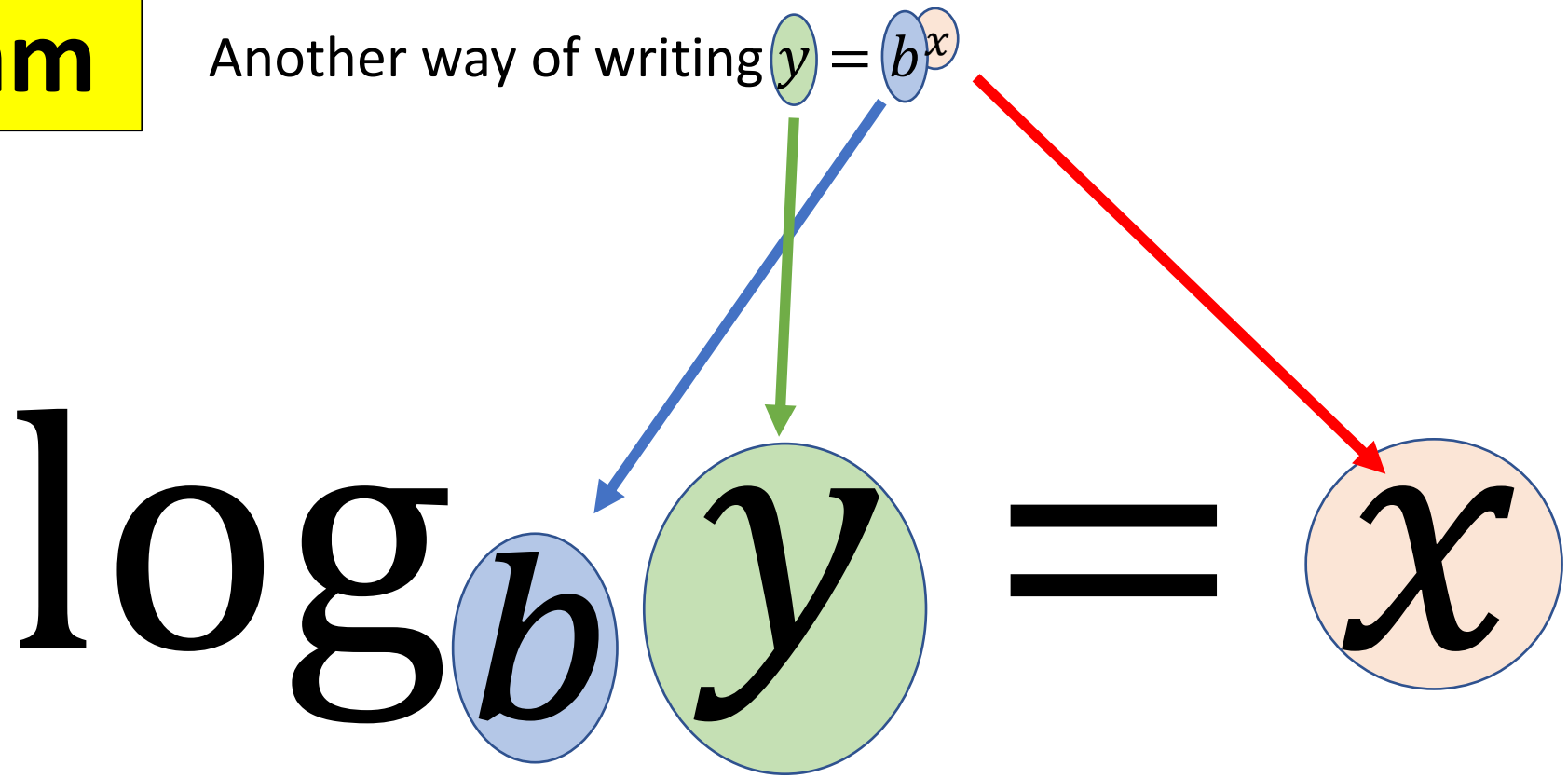
- Extraneous solution

Exponential Equation



Logarithm

Another way of writing $y = b^x$



Read it as "Log base b of y is x"

…as an exponential function it is b to the x is y

You will often see exponential equations in one of two forms

Exponent on each side, common base each side

Example: $3^x = 3^5$

- This is kind-of a duh…
- What is the only way this can be true?
  - since the bases are the same…
  - …the exponents have to be the same
  - So $x = 5$

What if the bases are not the same?

- Can you rewrite so they are the same?

Example: $3^x = 9^3$

- $9 = 3^2$
- So rewrite as $3^x = (3^2)^3$

$$3^x = 3^{2 \cdot 3}$$
$$3^x = 3^6$$
$$x = 6$$

Exponent only on 1 side

Example: $3^x = 5$

- Let's use $\log_b b^x = x$
- How?
  - Take the log of both sides…
  - …using the same base

$3^x = 5$ …base is 3

$\log_3 3^x = \log_3 5$ …log base 3 of each side

$x = \log_3 5$ …$\log_b b^x = x$

$x = \frac{\log 5}{\log 3}$ …change base

$x \approx 1.465$ …use calc

You will often see exponential equations in one of two forms

Exponent on each side, common base each side

Get bases the same…
…then the exponents must match

Exponent only on 1 side

1. Take the log of both sides…
   …using the same base
2. Use $\log_b b^x = x$ to get rid of log on 1 side
3. Change base
4. Use calculator

Solve each equation.

a. $100^x = \left(\frac{1}{10}\right)^{x-3}$ **b.** $2^x = 7$

SOLUTION

a.

$$100^x = \left(\frac{1}{10}\right)^{x-3}$$ Write original equation.

$$(10^2)^x = (10^{-1})^{x-3}$$ Rewrite 100 and $\frac{1}{10}$ as powers with base 10.

$$10^{2x} = 10^{-x+3}$$ Power of a Power Property

$$2x = -x + 3$$ Property of Equality for Exponential Equations

$$x = 1$$ Solve for x.

b.

$$2^x = 7$$ Write original equation.

$$\log_2 2^x = \log_2 7$$ Take $\log_2$ of each side.

$$x = \log_2 7$$ $\log_b b^x = x$

$$x \approx 2.807$$ Use a calculator.

Check

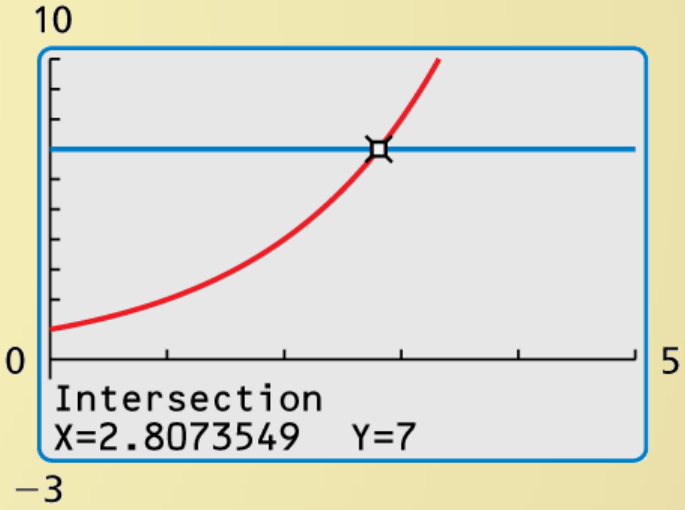
$$100^1 \stackrel{?}{=} \left(\frac{1}{10}\right)^{1-3}$$

$$100 \stackrel{?}{=} \left(\frac{1}{10}\right)^{-2}$$

$$100 = 100 \checkmark$$

Check

Enter $y = 2^x$ and $y = 7$ in a graphing calculator. Use the *intersect* feature to find the intersection point of the graphs. The graphs intersect at about (2.807, 7). So, the solution of $2^x = 7$ is about 2.807. ✓



Try it again, solve each equation.

a. $81^x = \left(\frac{1}{9}\right)^{x+3}$ **b.** $3^x = 8$

SOLUTION

a.

$$81^x = \left(\frac{1}{9}\right)^{x+3}$$ Write original equation.

$$(9^2)^x = (9^{-1})^{x+3}$$ Rewrite 81 and $\frac{1}{9}$ as powers with base 9.

$$9^{2x} = 9^{-x-3}$$ Power of a Power Property

$$2x = -x - 3$$ Property of Equality for Exponential Equations

$$x = -1$$ Solve for x.

b.

$$3^x = 8$$ Write original equation.

$$\log_3 3^x = \log_3 8$$ Take $\log_3$ of each side.

$$x = \log_3 8$$ $\log_b b^x = x$

$$x \approx 1.893$$ Use a calculator (also check on calc).

Check

$$81^{-1} \stackrel{?}{=} \left(\frac{1}{9}\right)^{-1+3}$$

$$\frac{1}{81} \stackrel{?}{=} \left(\frac{1}{9}\right)^2$$

$$\frac{1}{81} = \frac{1}{81} \checkmark$$

You are cooking *aleecha*, an Ethiopian stew. When you take it off the stove, its temperature is 212°F. The room temperature is 70°F, and the cooling rate of the stew is $r = 0.046$. How long will it take to cool the stew to a serving temperature of 100°F?

SOLUTION

Use Newton's Law of Cooling with $T = 100$, $T_0 = 212$, $T_R = 70$, and $r = 0.046$.

$T = (T_0 - T_R)e^{-rt} + T_R$	Newton's Law of Cooling
$100 = (212 - 70)e^{-0.046t} + 70$	Substitute for T, T_0, T_R, and r.
$30 = 142e^{-0.046t}$	Subtract 70 from each side.
$0.211 \approx e^{-0.046t}$	Divide each side by 142.
$\ln 0.211 \approx \ln e^{-0.046t}$	Take natural log of each side.
$-1.556 \approx -0.046t$	$\ln e^x = \log_e e^x = x$
$33.8 \approx t$	Divide each side by −0.046.

▶ You should wait about 34 minutes before serving the stew.

Homework

Pg 338, #1-20